

Chapter 10

CIRCLES

MODULE - 2/3



CIRCLES

In this module we learn about

- i) Perpendicular from the centre to a chord.
- ii) Circle through three points.
- iii) Equal chords and their distances from the centre.

Perpendicular from the centre to a chord.

Theorem 10.3: *The perpendicular from the centre of a circle to a chord bisects the chord.*

Given:- AB is a chord of the circle with centre O. $OM \perp AB$.

To prove:- $MA = MB$

Construction:- Join OA and OB.

Proof:- In triangles OAM and OBM

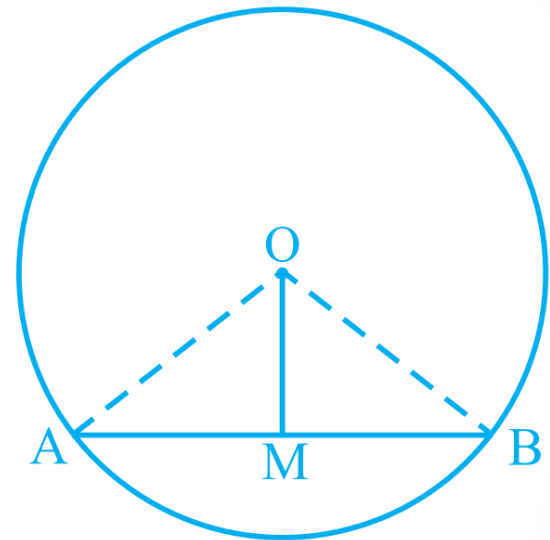
$$OA = OB \quad (\text{radii})$$

$$OM = OM \quad (\text{common})$$

$$\angle OMA = \angle OMB = 90^\circ \quad (OM \perp AB)$$

$$\triangle OAM \cong \triangle OBM \quad (\text{RHS rule})$$

$$MA = MB \quad (\text{CPCT})$$



Theorem 10.4: *The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.*

Given: AB is a chord of a circle with centre O and O is joined to the midpoint M of AB.

To prove : $OM \perp AB$.

Construction: Join OA and OB.

Proof: In triangles OAM and OBM

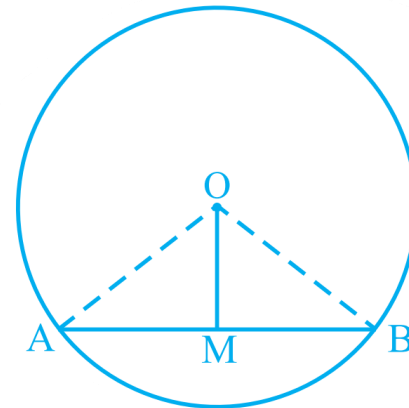
$$OA = OB \quad (\text{Why?})$$

$$AM = BM \quad (\text{Why?})$$

$$OM = OM$$

Therefore $\triangle OAM \cong \triangle OBM$ (HOW?)

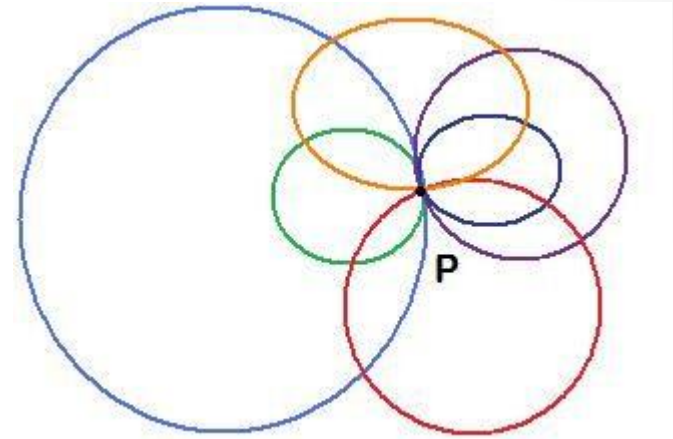
This gives $\angle OMA = \angle OMB = 90^\circ$. (CPCT)



Circle through three points.

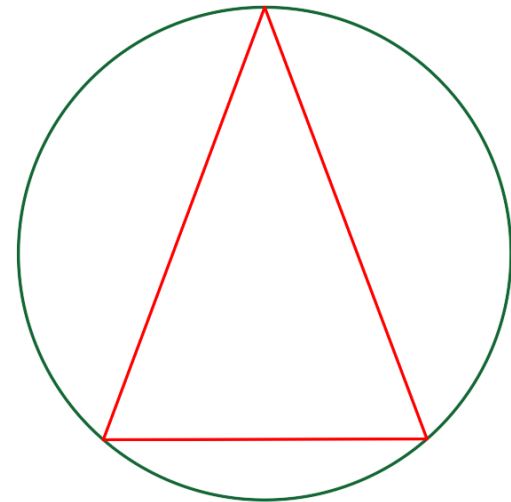
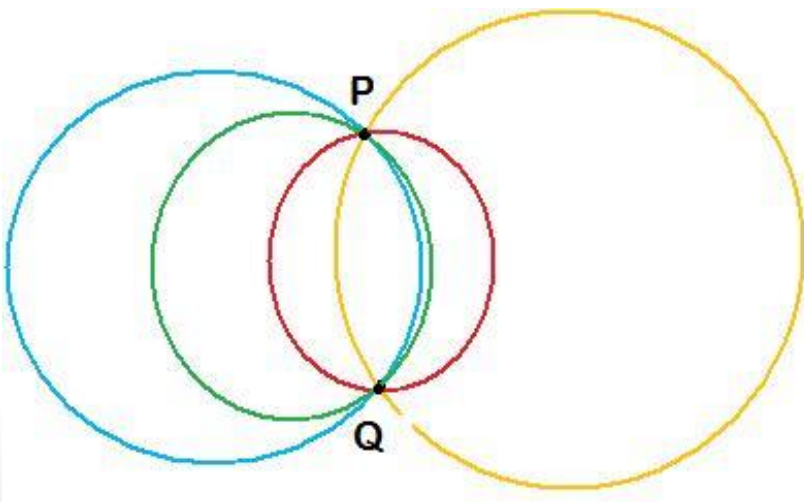
Circles through one point.

There can be many circles passing through one point or two points.



Circles through two points

Circle through three points



Theorem 10:5 *There is one and only one circle passing through three given non-collinear points.*

Let A,B and C be three non- collinear points. We can draw the perpendicular bisectors of AB and BC. Let these perpendicular bisectors intersect at a point P. (Why these perpendicular bisectors intersect?)

P lies on the perpendicular bisector of AB.

Therefore $AP=BP$ - - - 1

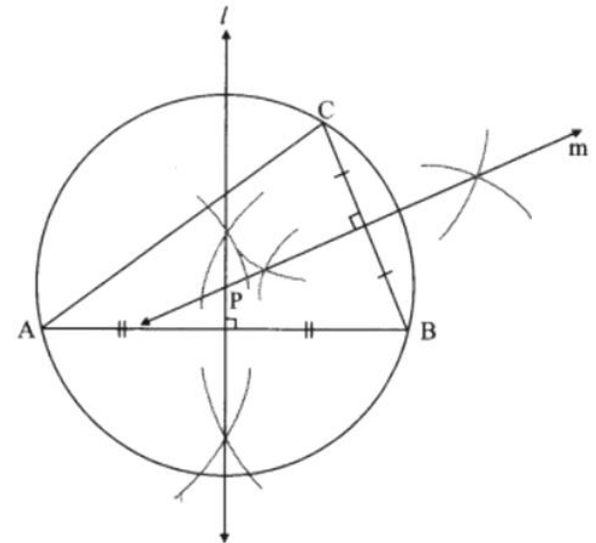
P lies on the perpendicular bisector of BC.

Therefore $BP=CP$ - - - 2

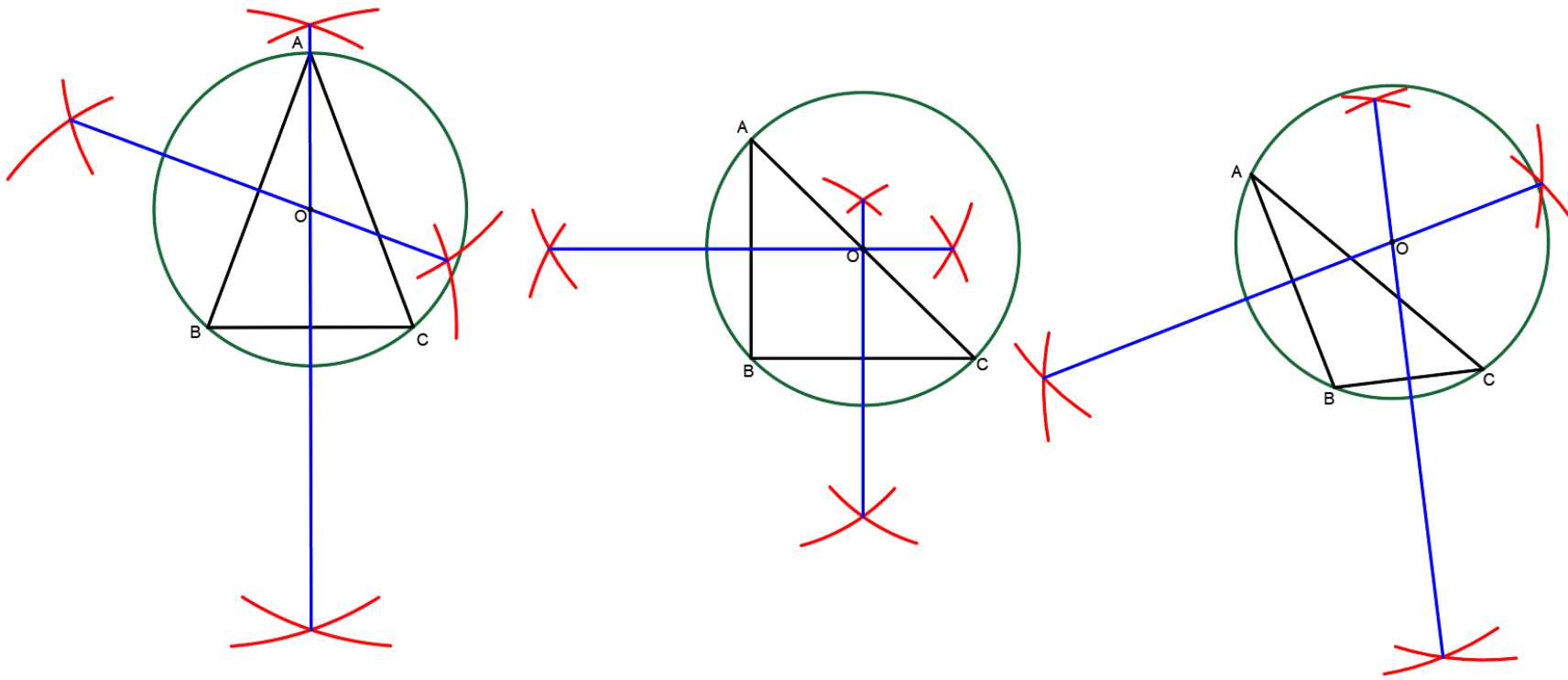
We get $AP=BP=CP$. (From 1 &2)

This shows that there is a circle passing through A,B and C with centre P and radius PA.

We know two lines (perpendicular bisectors) can intersect at only one point. So there is only one circle passing through the three points.



If ABC is a triangle, then there is a unique circle passing through the three vertices A, B and C of the triangle. This circle is called the circum-circle of the triangle ABC . Its centre and radius are called respectively the circumcentre and the circumradius of the triangle.



The Distance of a Line From a Point

The distance of a line from a point is the shortest distance between the point and the line.

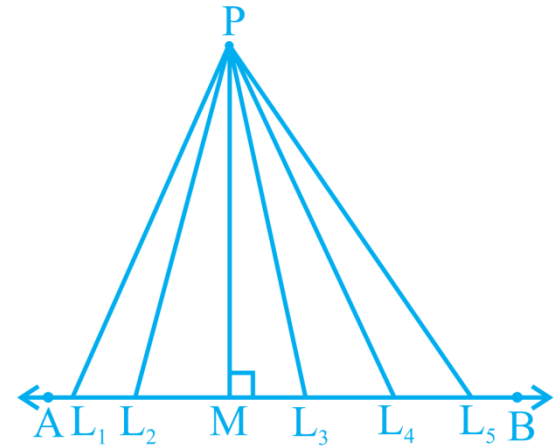
The perpendicular drawn from the point to the line is the shortest.

The length of the perpendicular from a point to a line is the distance of the line from the point.

If the point lies on the line, the distance of the line from the point is zero.

The distance between two intersecting lines is zero.

The distance between two parallel lines is the perpendicular distance between them.



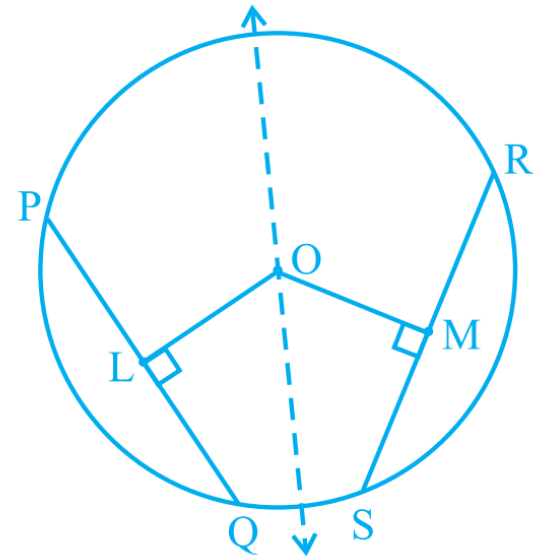
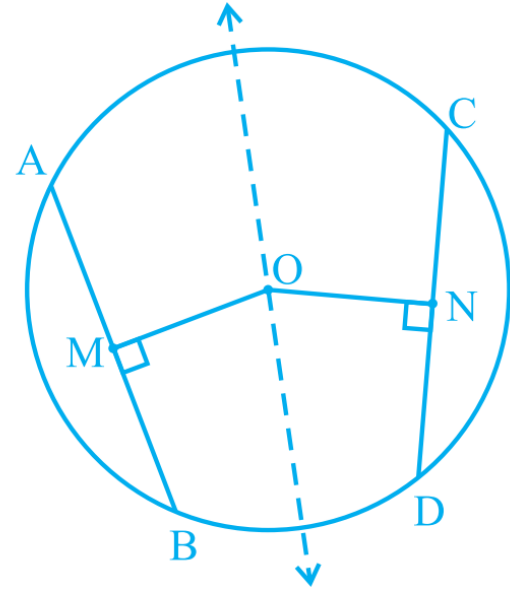
Equal Chords And Their Distance From The Centre.

Theorem 10.6: *Equal chords of a circle (or congruent circles) are equidistant from the centre (or centres)*

If $AB=CD$ then $OM=ON$

Theorem 10.7: *Chords equidistant from the centre of a circle are equal in length.*

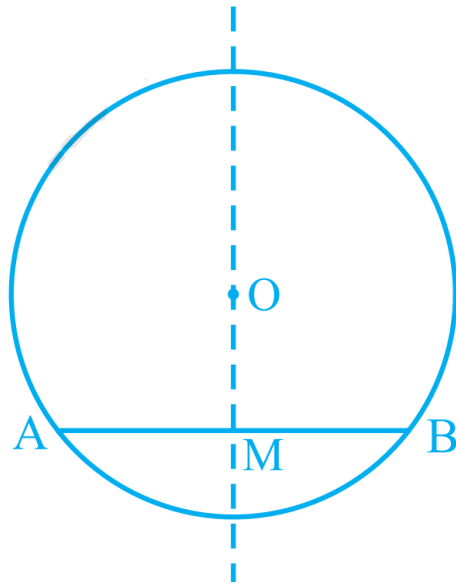
PQ and RS are two chords of a circle with centre O and $OL \perp PQ$ and $OM \perp RS$. If $OL=OM$, then $PQ=RS$.



Subject enrichment activities:

1. Draw a circle on a tracing paper. Let O be its centre. Draw a chord AB . Fold the paper along a line through O so that the portion of the chord falls on the other. Let the crease cut AB at the point M . Then OM is perpendicular to AB . And $MA = MB$

Give a proof yourself by joining OA and OB and proving the right triangles OMA and OMB to be congruent.



Subject enrichment activities

2. Draw a circum-circle to

- i) an acute angled triangle
- ii) a right angled triangle
- iii) an obtuse angled triangle

3. Draw a circle of any radius on a tracing paper.

Draw two equal chords AB and CD of it and also the perpendiculars OM and ON on them from the centre O . Fold the figure so that D falls on B and C falls on A . What do you observe? You observe that O lies on the crease and N falls on M .

Repeat the activity by drawing congruent circles with centres O and O' and taking equal chords AB and CD one on each. Draw perpendiculars OM and $O'N$ on them. Cut one circular disc and put it on the other so that AB coincides with CD . Write your observation.

